$\frac{u_{2n}}{u_{2n+1}} \geq 1$ car (u_n) est décroissante .

$\Rightarrow \frac{2n}{2n+1} \times \frac{u_{2n}}{u_{2n+1}} \geq \frac{2n}{2n+1}$

$\Leftrightarrow \frac{u_{2n+2}}{u_{2n+1}} \geq \frac{2n}{2n+1}$.

Ainsi $\frac{2n}{2n+1} \leq \frac{u_{2n+2}}{u_{2n+1}} \leq 1; \forall n \in IN$.

$$\left.\begin{array}{l} \forall n \in IN; \frac{2n}{2n+1} \leq \frac{u_{2n+2}}{u_{2n+1}} \leq 1 \\ \lim\limits_{n\to+\infty} \left[\frac{2n}{2n+1}\right] = \lim\limits_{n\to+\infty} \left[\frac{2}{2+1/n}\right] = 1 \end{array}\right\} \Rightarrow \lim_{n\to+\infty} \frac{u_{2n+2}}{u_{2n+1}} = 1$$

d- $\forall n \in IN$ on a :

$$\frac{u_{2n+2}}{u_{2n+1}} = \frac{\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} \times \ldots \times \frac{2n}{2n+1}}{\frac{1}{2} \times \frac{3}{4} \times \frac{5}{6} \times \ldots \times \frac{2n-1}{2n} \times \frac{\pi}{2}}$$

$$= \left(\frac{2 \times 4 \times 6 \times \ldots \times (2n)}{3 \times 5 \times \ldots \times (2n-1)}\right)^2 \times \frac{2}{(2n+1)\pi}$$

Comme $\lim\limits_{n\to+\infty} \frac{u_{2n+2}}{u_{2n+1}} = 1$

$$\Leftrightarrow \lim_{n\to+\infty} \left[\left(\frac{2 \times 4 \times 6 \times \ldots \times (2n)}{3 \times 5 \times \ldots \times (2n-1)}\right)^2 \times \frac{2}{2n+1} \times \frac{1}{\pi}\right] = 1$$

$$\Leftrightarrow \lim_{n\to+\infty} \left[\left(\frac{2 \times 4 \times 6 \times \ldots \times (2n)}{3 \times 5 \times \ldots \times (2n-1)}\right)^2 \times \frac{2}{2n+1}\right] = \pi.$$